

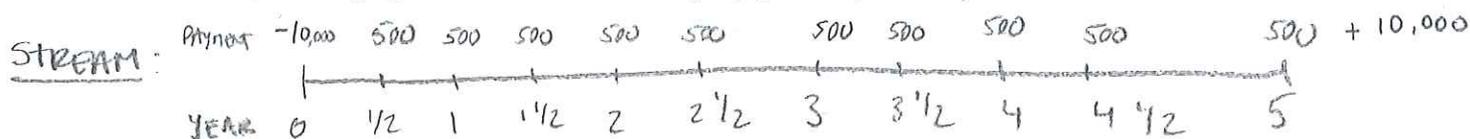
M451/551 Quiz 3

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1. A five-year \$10,000 bond with a 10% coupon rate costs \$10,000 and pays its holder \$500 every six months for five years, starting at the end of the sixth month, with a final additional payment of \$10,000 made at the end of those ten payments. Write down its present value if the (nominal yearly) interest rate is 6%. Assume the compounding is monthly. (You do not need to simplify your expression.)



$$PV(\text{STREAM}) = -10,000 + 500(\alpha)^6 + 500(\alpha^2)^6 + 500(\alpha^3)^6 + \dots + 500(\alpha^{10})^6 + 10,000(\alpha^{10})^6,$$

where $\alpha = \frac{1}{1 + \frac{0.06}{12}} = \frac{1}{1 + 0.005} = \frac{1}{1.005}$

Hence, the Present value is:

$$PV(\text{STREAM}) = -10,000 + \sum_{i=1}^{10} 500 (1.005)^{-6i} + 10,000 (1.005)^{-60}$$

+10

2. Show that the yield curve $\bar{r}(t)$ is a nondecreasing function of t if and only if

$$P(\alpha t) \geq (P(t))^\alpha \text{ for all } 0 \leq \alpha \leq 1, t \geq 0. \text{ Recall } P(t) = \exp\left(-\int_0^t r(s) ds\right).$$

(\Rightarrow) Suppose $\bar{r}(t)$ is a nondecreasing function of t .

this means: $\forall t_1, t_2$: If $t_1 \geq t_2$ then $\bar{r}(t_1) \geq \bar{r}(t_2)$

Now, note that $\bar{r}(t) = \frac{1}{t} \int_0^t r(s) ds$. Note that since

$\alpha \in [0, 1]$, $t \geq \alpha t$. It follows:

$\bar{r}(t) \geq \bar{r}(\alpha t)$, Moreover, e^x is an increasing function.

hence, $e^{\bar{r}(t)} \geq e^{\bar{r}(\alpha t)}$. But we can write

$$e^{\bar{r}(t)} = e^{\frac{1}{t} \int_0^t r(s) ds} = \left[e^{-\frac{1}{t} \int_0^t r(s) ds} \right]^\alpha \times [P(t)]^\alpha$$

$$e^{\bar{r}(\alpha t)} = e^{\frac{1}{\alpha t} \int_0^{\alpha t} r(s) ds} = e^{-\frac{1}{\alpha t} \int_0^{\alpha t} r(s) ds} = P(\alpha t)$$

Therefore, $P(\alpha t) \geq (P(t))^\alpha$, which follows because e^{-x} is decreasing for $x \geq 0$.

(\Leftarrow) Note that this direction follows

just from reading the previous proof

from bottom to top.

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